COURSE 02457

Signal Processing in Non-linear Systems: Lecture 1

- The ISP group
- Course overview
- Neuroimaging
- What's in the future for signal processing?
- Probabilities and densities
- Conditional probabilities and densities
- Bayes' theorem

"Nothing is more practical than a good theory"
Vladimir Vapnik (Russian statistician)

Conditional probabilities, Bayes' theorem

Figure 1: The measured signals take (discrete) values X and each signal is assigned to one of the classes $\mathcal{C}_1, \mathcal{C}_2$. The number of dots in each cell corresponds to the number of signals that fall in the given class and have taken on the value X

$$
\begin{aligned}
P(\mathcal{C}_k, X^l) &= P(X^l|\mathcal{C}_k)P(\mathcal{C}_k) \\
P(\mathcal{C}_k, X^l) &= P(\mathcal{C}_k|X^l)P(X^l)
\end{aligned}
$$

$$
\begin{aligned}
P(\mathcal{C}_k|X^l) &= \frac{P(X^l|\mathcal{C}_k)P(\mathcal{C}_k)}{P(X^l)} \\
P(X^l|\mathcal{C}_k) &= \frac{P(\mathcal{C}_k|X^l)P(X^l)}{P(\mathcal{C}_k)}
\end{aligned}
$$

Conditional probabilities cont'd

$$
\begin{aligned}
P(\mathcal{C}_1, |X^l) + P(\mathcal{C}_2|X^l) &= 1 \\
\frac{P(X^l|\mathcal{C}_1)P(\mathcal{C}_1)}{P(X^l)} + \frac{P(X^l|\mathcal{C}_2)P(\mathcal{C}_2)}{P(X^l)} &= 1 \\
P(X^l|\mathcal{C}_1)P(\mathcal{C}_1) + P(X^l|\mathcal{C}_2)P(\mathcal{C}_2) &= P(X_l)
\end{aligned}
$$

Conditional probabilities cont'd

Figure 2: Schematic plot of the histograms for a measured signal drawn from either of two populations $\mathcal{C}_1, \mathcal{C}_2$

Figure 3: Corresponding $P(X)$

Figure 4: Corresponding $P(\mathcal{C}|X)$'s

Bayes' theorem – multivariate version

$$
\begin{aligned}
P(\mathcal{C}_k, \mathbf{x}) &= p(\mathbf{x}|\mathcal{C}_k)P(\mathcal{C}_k) \\
P(\mathcal{C}_k, \mathbf{x}) &= P(\mathcal{C}_k|\mathbf{x})p(\mathbf{x})
\end{aligned}
$$

$$
\begin{aligned}
P(\mathcal{C}_k|\mathbf{x}) &= \frac{p(\mathbf{x}|\mathcal{C}_k)P(\mathcal{C}_k)}{p(\mathbf{x})} \\
p(\mathbf{x}|\mathcal{C}_k) &= \frac{P(\mathcal{C}_k|\mathbf{x})p(\mathbf{x})}{P(\mathcal{C}_k)}
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k=1}^{c} P(\mathcal{C}_k|\mathbf{x}) &= 1 \\
\sum_{k=1}^{c} p(\mathbf{x}|\mathcal{C}_k)P(\mathcal{C}_k) &= p(\mathbf{x})
\end{aligned}
$$